Periodic waves and solitons in a nonlinear fibre with resonant impurities

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Abstract. We shall consider a coupled nonlinear Schrödinger equation–Bloch system of equations describing the propagation of a single pulse through a nonlinear dispersive waveguide in the presence of resonances; this could be, for example, a doped optical fibre. By making use of the integrability of the dynamic equations, we shall apply the finite-gap integration method to obtain periodic solutions for this system. Next, we consider the problem of the formation of solitons at a sharp front pulse and, by means of the Whitham modulational theory, we derive the amplitude and velocity of the largest soliton.

1. Introduction

Until recently the investigation of optical solitons has been classified into two main categories which distinguish two regimes: resonant propagation, related to self-induced transparency (SIT) and non-resonant propagation, connected to phenomena described by the nonlinear Schrödinger equation (NLS). In both cases, solitons may be present in the systems, as the equations describing them turn out to be integrable. The existence of a mixed soliton, that is a simultaneous soliton solution of the coupled system NLS–Bloch equations, has been considered in [1, 2] and investigated numerically in two-level [3, 4] and three-level [5] atomic media. In these cases, solitons may appear provided that a certain balance condition is satisfied. These investigations shed light on several new possibilities that the mixed state offers, useful for later applications on novel optical techniques, such as pulse cloning and tailoring. The combination of population trapping states with dispersive Kerr effects has exhibited a good potential as a scenario for new phenomena by creating suitable conditions for smooth propagation. The resonant
light pulse, instead of being absorbed, propagates freely as if the SIT effect had paved the way for the soliton passage. Doped optical fibres are good candidates for practical realizations of a two-level atomic system embedded in a dispersive nonlinear host whose waveguiding properties avoids diffraction transverse effects. There is much interest in erbium-doped fibres for optical communication properties but other rare-earth elements such as neodymium and samarium may also be used. Fibre fabrication satisfying particular specifications could prove to be an ideal medium to reproduce the spectacular experiments reported in vapour media recently [6], where the atomic system is manipulated by electromagnetic fields. Another possibility is semiconductor microresonators with embedded organic thin films (quantum wells) which permit one to enhance nonlinear response coefficients and control relative strength of Kerr-like third order nonlinearity and two-level nonlinearity [7].

In this paper, we shall consider the coupled NLS–Bloch system of equations describing the propagation of a single pulse through a nonlinear dispersive waveguide in the presence of resonances; this could be, for example, a doped optical fibre. In this case the resonances are provided by the dopant. We shall tackle two interconnected problems, namely obtaining periodic solutions for this problem and studying the formation of a train of solitons from a step-like initial condition, that is predicting the parameters of the highest soliton. The context of this is to assume that the system under consideration is described by the integrable case of the Maxwell–Bloch equation. This represents such a balance between resonant effects and the Kerr effect that the soliton solution can be presented in a simple analytical form. It should be noted, however, that the results are robust with respect to perturbations out of integrability. The overall picture remains valid in the near-integrable case. With this in mind, we shall therefore apply the finite-gap integration method, based on the inverse spectral transform (IST) method and find periodic waves and solitons solutions for the NLS–Bloch system of equations representing the interaction of a single field with a two-level system. In this way we hope to find SIT–NLS analogues of the results established in the literature for the SIT component, and which have been experimentally confirmed [8]. Furthermore, as occurs with three-level media and two matched fields [9] where the problem is equivalent to two level media interacting with one field, we believe that the present solutions may also help us to understand the results for the mixed SIT–NLS solitons in three-level media.

2. Periodic waves and solitons

Therefore let us consider propagation along the $Z$ direction of an electromagnetic field linearly polarized in the $X$ direction with a slowly varying envelope $E(T,Z)$ through the medium, which is composed of a dispersive nonlinear waveguide upon which two-level atoms have been evenly distributed. In the following we shall neglect relaxation effects by supposing that the main effects under consideration (e.g. the duration of soliton pulses) are shorter than the atomic dephasing time. Let $\Delta$ be the frequency offset parameter of the atomic transitions from the oscillation frequency of the electromagnetic field of the wave, and $d$ and $n$ the dipole moment of transitions (polarization) and the population of the atoms respectively. The equation describing this systems is (see, for example, [4])
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\[ iE_Z + E_{TT} + 2a^2|E|^2E + d = 0, \]
\[ d_T + 2i\Delta d = -2iaE_n, \]
\[ n_T = ia(d^*E - dE^*). \]

Furthermore, \( d \) and \( n \) are related by the normalization condition
\[ |d|^2 + n^2 = 1, \]
which reflects the conservation of probability; the total probability that an atom can be found in the upper or lower level is equal to unity. We neglect here the inhomogeneous line broadening and assume that all atoms have the same frequency shift. The system (1) is written in the variables
\[ Z = z, \quad T = t - z, \]
\( z \) and \( t \) being the dimensionless spatial coordinate and time respectively. The coupling constants of nonlinear Kerr self-action \((a^2)\) and of interaction with resonant impurities \((a)\) are adjusted so that equation (1) satisfies the complete integrability condition [1], and hence we can apply the IST method to them.

The IST method is based on the existence of a Lax pair, that is on the possibility of representing the system (1) as a compatibility condition of two linear systems with spectral parameter \( \lambda \):
\[
\begin{pmatrix}
\psi_1 \\
\psi_2
\end{pmatrix}_T =
\begin{pmatrix}
F & G \\
H & -F
\end{pmatrix}
\begin{pmatrix}
\psi_1 \\
\psi_2
\end{pmatrix},
\begin{pmatrix}
\psi_1 \\
\psi_2
\end{pmatrix}_Z =
\begin{pmatrix}
A & B \\
C & -A
\end{pmatrix}
\begin{pmatrix}
\psi_1 \\
\psi_2
\end{pmatrix},
\]
where
\[ F = -i\lambda, \quad G = -iaE, \quad H = -iaE^*, \]
and
\[
\begin{align*}
A &= -2i\lambda^2 + ia^2|E|^2 + \frac{ia}{2\lambda - \lambda}, \\
B &= -ia\left(2\lambda E + iE_T + \frac{d}{2(\lambda - \lambda)}\right), \\
C &= -ia\left(2\lambda E^* - iE_T^* + \frac{d^*}{2(\lambda - \lambda)}\right).
\end{align*}
\]
The \( Z \) evolution of eigenfunctions \( \psi = (\psi_1, \psi_2) \) is described by the well-known Zakharov–Shabat problem common for the NLS equation and SIT system, whereas \( T \) evolution of \( \psi \) corresponds to some combination of \( \mathbf{V} \) matrices of these two integrable systems.

Following the well-known finite-gap method of integration (see, for example, [10]), we define the ‘squared basis functions’
\[
f = -\frac{i}{2}(\psi_1\phi_2 + \psi_2\phi_1), \quad g = \psi_1\phi_1, \quad h = -\psi_2\phi_2,
\]
built from two sets of basis solutions \((\psi_1, \psi_2)\) and \((\phi_1, \phi_2)\) of the linear systems (4). The functions (7) satisfy the linear systems
\[
f_T = -iHg + iGh, \quad g_T = 2iGf + 2Fg, \quad h_T = -2iHf - 2Fh,
\]
and
\[
f_Z = -iCg + iBh, \quad g_Z = 2iBf + 2Ag, \quad h_Z = -2iCf - 2Ah,
\]
which have an integral of motion

\[ f^2 - gh = P(\lambda). \]  \hspace{1cm} (10)

The periodic solutions of the initial system (1) are distinguished by the condition that \( P(\lambda) \) is a polynomial in the spectral parameter \( \lambda \). The simplest and most important for physical applications case is the one-phase periodic wave which corresponds to a fourth-degree polynomial

\[
P(\lambda) = \prod_{i=1}^{4}(\lambda - \lambda_i)
= \lambda^4 - s_1\lambda^3 + s_2\lambda^2 - s_3\lambda + s_4
= \prod_{i=1}^{4}[(\lambda - \Delta) - (\lambda_i - \Delta)]
= (\lambda - \Delta)^4 - \delta_{1}(\lambda - \Delta)^3 + \delta_{2}(\lambda - \Delta)^2 - \delta_{3}(\lambda - \Delta) + \delta_{4}.
\]  \hspace{1cm} (11)

We look for the solution of equations (8) and (9) in the form

\[
f(\lambda) = \lambda^2 - \Delta^2 - f_1(\lambda - \Delta) + f_2,
g(\lambda) = -iaE(\lambda - \mu),
h(\lambda) = -iaE^*(\lambda - \mu^*).
\]  \hspace{1cm} (12)

Then substitution of equations (12) into equations (8) and (9) gives

\[
f_{1,T} = f_{1,Z} = 0, \quad \text{that is} \quad f_1 = \text{constant},
f_{2,T} = -ia^2|E|^2(\mu - \mu^*), \quad f_{2,Z} = \frac{a}{2}n_T + 2f_1 f_{2,T},
f_2 = C\alpha,
\mu = \Delta + \frac{Cd}{E}, \quad \mu^* = \Delta + \frac{Cd^*}{E^*},
\]  \hspace{1cm} (13)-(15)

where \( C \) is an as yet unknown real constant factor, and

\[
iE_{T} = -2E(\mu - f_1),
iE_{Z} = 4E(\Delta^2 - f_1\Delta - f_2) - 4f_1E(\mu - f_1) - d - 2a^2|E|^2E.
\]  \hspace{1cm} (16)

To find the values of \( f_1 \) and \( C \), we substitute equations (11) and (12) into equation (10) and compare the coefficients of equal degrees of \( \lambda - \Delta \) to obtain the system

\[
2(f_1 - 2\Delta) = \delta_{1} = s_1 - 4\Delta,
(f_1 - 2\Delta)^2 + 2C\alpha + a^2|E|^2 = \delta_{2},
2C\alpha(f_1 - 2\Delta) + a^2|E|^2(\mu + \mu^* - 2\Delta) = \delta_{3},
(C\alpha)^2 + a^2|E|^2(\mu - \Delta)(\mu^* - \Delta) = \delta_{4}.
\]  \hspace{1cm} (17)-(18)

The first equation gives at once

\[
f_1 = \frac{1}{2}s_1.
\]  \hspace{1cm} (19)
whereas the last equation reduces to equation (2) provided that \( C^2 = \frac{\hat{s}_4}{a^2} \). We choose the sign as \( C = \frac{-\hat{s}_4}{a} \) to make \( n = -1 \) for the ground state of atoms without an electromagnetic wave (see below). Thus, we have

\[
f_2 = -\frac{\sqrt{\hat{s}_4}}{2} n = -[P(A)]^{1/2} n, \quad \mu = \Delta - \frac{[P(A)]^{1/2} d}{a E}.
\]  

(20)

The second equation of equations (18) gives the relation between \( n \) and \(|E|^2\):

\[
n = \frac{1}{2[\frac{P(A)}{2}]^{1/2}} (a^2|E|^2 - \hat{s}_2 + \frac{1}{\lambda_1^2}).
\]  

(21)

The last two equations of equations (18) permit us to express \( \mu \) and \( \mu^* \) as functions of the intensity

\[\nu = a^2|E|^2.\]  

(22)

Simple calculation (see [10]) gives

\[
\mu = \frac{s}{4} - \frac{q + i[-\mathcal{R} ]^{1/2}}{2\nu},
\]  

(23)

where \( \mathcal{R} = \nu^3 - 2p\nu^2 + (p^2 - 4r)\nu + q^2 \),

(24)

and the parameters \( s, p, q \) and \( r \) are expressed in terms of the coefficients of \( P(\lambda) \):

\[
s = s_1, \\
p = s_2 - \frac{3\hat{s}_1^2}{8s_1}, \\
q = \frac{1}{8s_1}(s_2 - \frac{3\hat{s}_1^2}{4}) - s_3, \quad \text{(25)}
\\r = s_4 + \frac{1}{2s_1}(s_2 - \frac{3\hat{s}_1^2}{16}) - \frac{1}{8s_1}s_3.
\]

The zeros \( \nu_i, i = 1, 2, 3 \), of \( \mathcal{R} \) are related to the zeros \( \lambda_i, i = 1, 2, 3, 4 \), of the polynomial \( P(\lambda) \) by the simple symmetric formulae

\[
\nu_1 = -\frac{1}{4}(\lambda_1 - \lambda_2 + \lambda_3 - \lambda_4)^2, \\
\nu_2 = -\frac{1}{4}(\lambda_1 - \lambda_2 - \lambda_3 + \lambda_4)^2, \quad \text{(26)}
\\\nu_3 = -\frac{1}{4}(\lambda_1 + \lambda_2 - \lambda_3 - \lambda_4)^2.
\]

As we see from equations (14) and (20), \( n \) and \( \nu \) depend on \( T \) and \( Z \) only through the phase

\[
\xi = T + \left(s_1 - \frac{a}{2[\frac{P(A)}{2}]^{1/2}}\right)Z, \\ \text{(27)}
\]

and with the use of equation (23) we find the equation for \( \nu \):

\[
\frac{d\nu}{d\xi} = 2[-\mathcal{R} ]^{1/2}. \
\]  

(28)

The variable \( \nu \) is positive by definition and, hence, it can only oscillate between two positive zeros of \( \mathcal{R} \). Therefore the zeros \( \lambda_i \) of \( P(\lambda) \) must have the form

\[
\lambda_1 = \alpha + i\gamma, \quad \lambda_2 = \beta + i\delta, \quad \lambda_3 = \alpha - i\gamma, \quad \lambda_4 = \beta - i\delta, \quad \text{(29)}
\]
and then from equations (26) we obtain
\[ \nu_1 = -(\alpha - \beta)^2, \quad \nu_2 = (\gamma - \delta)^2, \quad \nu_3 = (\gamma + \delta)^2, \]
that is \( \nu \) oscillates between \( \nu_2 \) and \( \nu_3 \). The solution of equation (28) is readily expressed in terms of elliptic functions, which gives a simple expression for the intensity of light:
\[ \nu = a^2 |E(T, Z)|^2 = \nu_3 + (\nu_2 - \nu_3) \text{sn}^2([\nu_3 - \nu_1]^{1/2} \xi, m) = (\gamma + \delta)^2 - 4\gamma \delta \text{sn}^2([(\alpha - \beta)^2 + (\gamma + \delta)^2]^{1/2} \xi, m), \]
where the parameter \( m \) of the elliptic function is given by
\[ m = \frac{\nu_3 - \nu_2}{\nu_3 - \nu_1} = \frac{(\lambda_1 - \lambda_3)(\lambda_2 - \lambda_4)}{(\lambda_1 - \lambda_4)(\lambda_2 - \lambda_3)} = \frac{4\gamma \delta}{(\alpha - \beta)^2 + (\gamma + \delta)^2}. \]
From equation (17) with the use of equations (20) and (21) we find that
\[ E(T, Z) = \exp \left[ -i\xi T + i \left( -12A^2 + 6s_1 A - \frac{3}{2}s_1^2 + 2s_2 + \frac{aA}{[P(A)]^{1/2}} \right) Z \right], \]
where \( \tilde{E}(T, Z) \) depends only on the phase \( \xi \) and satisfies the equation
\[ \frac{d\tilde{E}}{d\xi} = 2i \mu \tilde{E}. \]
It can be solved in the same way as was done for the NLS equation [11] and SIT system [12], and the result is expressed in terms of Weierstrass elliptic functions, so that
\[ E(T, Z) = -\frac{2}{a} \exp \left[ -i\xi T + i \left( -12A^2 + 6s_1 A - \frac{3}{2}s_1^2 + 2s_2 + \frac{aA}{[P(A)]^{1/2}} \right) Z + \frac{i\xi}{2} - 2\zeta(\kappa)\xi - \eta'\kappa \right] \frac{\sigma(\kappa + \omega' + 2\xi)}{\sigma(\omega' + 2\xi)\sigma(\kappa)}, \]
where \( \kappa \) is determined by the relation
\[ \varphi(\kappa) = \frac{1}{6} \rho, \]
\( \omega \) and \( \omega' \) are half-periods of \( \varphi(\xi) \), and \( \eta' = \zeta(\omega') \).
In the soliton limit we have
\[ \lambda_1 = \lambda_2 = \alpha + i\gamma, \quad \lambda_3 = \lambda_4 = \alpha - i\gamma, \]
and well-known limiting expressions for elliptic functions yield
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\[ E(T, Z) = \frac{2\gamma}{a} \exp \left[ -2i\alpha T - i \left( 4(\alpha^2 - \gamma^2) + \frac{a(\alpha - \Delta)}{(\alpha - \Delta)^2 + \gamma^2} \right) Z \right] \times \frac{1}{\cosh \left[ 2\gamma(T + Z/V) \right]} \]

\[ d = -\frac{2\gamma}{(\alpha - \Delta)^2 + \gamma^2} \frac{\alpha - \Delta + i\gamma \tanh \left[ 2\gamma(T + Z/V) \right]}{\cosh \left[ 2\gamma(T + Z/V) \right]} \times \exp \left[ -2i\alpha T - i \left( 4(\alpha^2 - \gamma^2) + \frac{a(\alpha - \Delta)}{(\alpha - \Delta)^2 + \gamma^2} \right) Z \right], \]

\[ n = \frac{2\gamma^2}{(\alpha - \Delta)^2 + \gamma^2 \cosh \left[ 2\gamma(T + Z/V) \right]} - 1, \]

where

\[ \frac{1}{V} = 4\alpha - \frac{a}{2[(\alpha - \Delta)^2 + \gamma^2]}. \]

As one should expect, this solution combine properties of NLS and SIT solitons. The levels population \( n \) goes to \(-1\) as \( |T, Z| \to \infty \), which justifies the choice of the sign in equation (20).

3. Formation of solitons at a sharp front of the pulse

When a long enough pulse with a sharp front enters the nonlinear medium, oscillations arise at the front owing to modulational instability of the pulse, and these oscillations evolve gradually into a modulated periodic wavetrain with solitons at one of its ends and small amplitude oscillations at the other end. Such an evolution can be described by the Whitham [13] modulation theory [10], and we shall apply this method here to the periodic solution found in the preceding section. This approach is correct even for long enough pulses provided that their fronts are sharp enough and the characteristic time of formation of solitons at the sharp front is shorter than the relaxation times. If these conditions are satisfied, then such idealizations as rectangular or step-like pulses can be used for analytical estimates.

In the modulated periodic wave the parameters \( \lambda_i, i = 1, 2, 3, 4 \), become slow functions of \( Z \) and \( T \) which change little in one wavelength and one period. Their slow evolution is described by the Whitham equations which can be written in our case in the form [10]

\[ \frac{\partial \lambda_i}{\partial Z} + \frac{1}{v_i} \frac{\partial \lambda_i}{\partial T} = 0, \quad i = 1, 2, 3, 4, \]

where

\[ \frac{1}{v_i} = \left( 1 + \frac{\Omega}{\lambda_i \Omega_i} \right) \frac{1}{V}, \quad \lambda_i = \frac{\partial}{\partial \lambda_i}, \quad i = 1, 2, 3, 4, \]

\( V \) is the phase velocity of the wave (see equation (27)), and \( \Omega \) is the frequency of the wave which is expressed up to an inessential constant factor by the formula

\[ \frac{2\pi}{\Omega} = \int \frac{d\mu}{[P(\mu)]^{1/2}} \propto \frac{K(m)}{[(\lambda_1 - \lambda_4)(\lambda_2 - \lambda_3)]^{1/2}}, \]
$K(m)$ being the complete elliptic integral of the first kind.

We suppose for simplicity that the initial pulse has a step-like form as a function of $T$ along the entire $T$ axis.

$$|E(T,Z)| = \begin{cases} |\gamma/a|, & T > 0, \\ 0, & T < 0, \end{cases}$$

(45)

and $d$ and $n$ are given by the corresponding piecewise plane-wave solutions of equations (1). Since these initial conditions do not have any parameter with the dimension of length or time, we can suppose that $\lambda_i$ depend only on the self-similar variable $\zeta = Z/T$. Because $\lambda_3 = \lambda_1^*$ and $\lambda_4 = \lambda_2^*$, it is sufficient to use only two Whitham equations (42) which in our self-similar case take the form

$$\frac{d\lambda_1}{d\zeta}(v_1 - \zeta) = 0, \quad \frac{d\lambda_2}{d\zeta}(v_2 - \zeta) = 0.$$  

(46)

It is easy to see that the solution of these equations corresponding to our initial data is given by

$$\lambda_1 = \alpha + i\gamma = \text{constant},$$

(47)

and $v_2 = \zeta$.

(48)

Separating here real and imaginary parts, we find expressions defining in implicit form the dependences of $\beta$ and $\delta$ on $\zeta$. Thus, the parameters $\alpha, \gamma, \beta(Z/T)$ and $\delta(Z/T)$ are determined, and their substitution into equation (31) or (35) for periodic solution gives expressions for the electromagnetic field in the region of oscillations on the $T$ axis for all $Z > 0$. The resulting picture coincides qualitatively with those for NLS and SIT cases [10] apart from more complicated expression for the phase velocity of the wave. Therefore we shall not give here a full analysis of the solution obtained and note only that at the solitonic end of the region of oscillations, where $\beta = \alpha$ and $\delta = \gamma$, the periodic wave reduces to the soliton solution (38)–(40). Hence, the intensity of the ‘first’ soliton is four times the intensity of the initial step-like pulse (45), and this soliton moves with the velocity (41). These predictions of the Whitham theory agree quite well with direct numerical solution of equations (1).

In figures 1 and 2 the intensity $|E(T,Z)|^2$ is shown as a function of $T$ for two fixed values of $Z$; figure 1 corresponds to $Z = 3$ and figure 2 to $Z = 4$. These plots illustrate the time dependence of intensity at different positions for an initial step-like pulse switched on at the moment $T = 0$ at the point $Z = 0$. The parameters of the pulse are chosen as $\alpha = -1$, $\gamma = 1$ and $\Delta = 0.3$. It is seen that the maximum intensity of the first soliton is quite close to the theoretical value $(2\gamma)^2 = 4$, although the asymptotic state has not been totally reached yet. The mean velocities of solitons for these quite short periods differ a little from the value $V_{\text{theor}} = 0.22$ given by equation (41); for the case $Z = 3$, the mean velocity $V_{\text{mean}} = 0.19$ and, for $Z = 4$, $V_{\text{mean}} = 0.20$. This discrepancy can be explained by the influence of the initial stage of formation of solitons which is not taken into account in the Whitham asymptotic theory. This explanation is confirmed by the fact that the ‘instantaneous’ velocity $V_{\text{instant}}$ of the first soliton for the interval from $Z = 3$ to $Z = 4$ is equal to 0.22, in excellent agreement with the theoretical value. The non-monotonic dependence of soliton amplitudes in figure 2 can be explained as a result of competition of two tendencies; for purely Kerr nonlinearity the first
soliton has the maximum amplitude whereas for the purely SIT case the amplitude of solitons first increases, next reaches the maximum value and then decreases (see, for example, [10]). Thus, the Whitham theory gives a simple description of the process of formation of solitons at a sharp front and predicts quite well the parameters of the first soliton.

4. Conclusion
In conclusion, we have investigated the periodic and soliton solutions of the coupled NLS–Bloch system for a choice of the parameters such that this system is
completely integrable. The results obtained must be robust with respect to variations in these parameters and can be applied, at least qualitatively, to nearly integrable cases, too. Experiments on observations of self-induced transparency performed on fibres doped with 8990 ppm erbium have demonstrated how delicate the problem of accomplishing the integrability condition is [14, 15]. For example, using typical values for the nonlinear Kerr index and the dipole moment in silica-based erbium-doped fibres and considering that, at room temperature, the homogeneous lifetime $T_r$ of the atomic system is shorter than 1 ps, one finds that $P_{SIT} = 2.7$ GW, a power six orders of magnitude larger than that required for the NLS soliton. This extremely high coupled power hinders the observation of ‘mixed’ SIT–NLS solitons in erbium-doped fibres. However, $P_{SIT}$ is inversely proportional to the pulse duration $\tau$ so that, by lowering the temperature, one may prolong $T_r$ and consequently the pulse duration $\tau$. Lowering the temperature is then one way of reducing the coupled power. A study developed on the optical dephasing rates of a dilute concentration of Nd$^{3+}$ ions in a pure silica fibre has been performed showing that, at a temperature as low as 4.2 K, $T_r$ is of the order of 10 ns [16]. By cooling the fibre to this temperature, SIT solitons were observed in an erbium fibre 1.5 m long. It would be of considerable interest to observe experimentally the phenomena exhibited in numerical simulations of SIT–NLS soliton propagation in doped fibres such as cloning. We believe that a thorough investigation of fibre characteristics should indicate the most promising media for such observation.

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